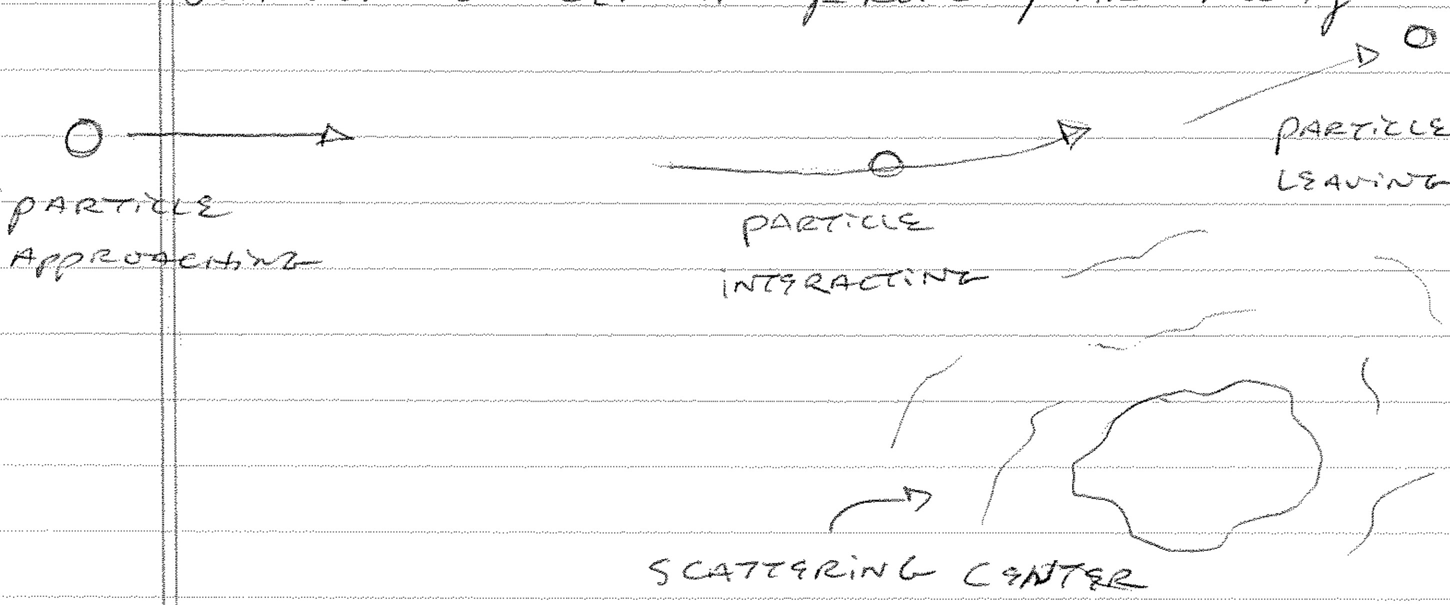
(1)

PERTURBATION THEORY ELSEWHERE IN PHYSICS

TO INCREASE YOUR COMFORT/FAMILIARITY WITH PERTURBATION THEORY, WE WILL LOOK AT ~~HOW~~ SOME EXAMPLES OF P.T. IN OTHER AREAS OF PHYSICS. HOPEFULLY YOU WILL GET AN IDEA OF ~~THE~~ WHAT UNIFIES ALL THESE FAIRLY DISTINCT SCENARIOS: ONE PREDOMINANT INFLUENCE GENERATI~~NG~~ A PRIMARY RESPONSE WHICH IN ~~THE~~ TURN ~~EXPERIENCE~~ IS SUBJECTED TO A SECONDARY INFLUENCE WHICH GENERATES A SECONDARY RESPONSE, SO ON & SO FORTH...

(1A) CLASSICAL SMALL-ANGLE SCATTERING

IMAGINE A PARTICLE APPROACHING A SCATTERING CENTER FROM FAR AWAY:



IF THE PARTICLE IS MOVING FAST AND IS NOT AIMED DIRECTLY AT THE SCATTERER, OUR INTUITION TELLS US THAT THE PARTICLE WILL ONLY BE DEFLECTED BY A SMALL ANGLE. IN OTHER WORDS, ITS MOTION WILL NOT BE SIGNIFICANTLY DIFFERENT HAD THERE BEEN NO SCATTERER AT ALL. THAT IS THE KEY BEHIND THIS APPROXIMATION.

1Ai WHAT IS THE MOTION, FOR ALL TIMES, OF A PARTICLE APPROACHING FROM A POINT $\vec{x}_0$, MOVING WITH A VELOCITY $\vec{v}$? ASSUME NO FORCES ACT ON PARTICLE.

i.e. $\vec{x}(t) = \qquad ?$

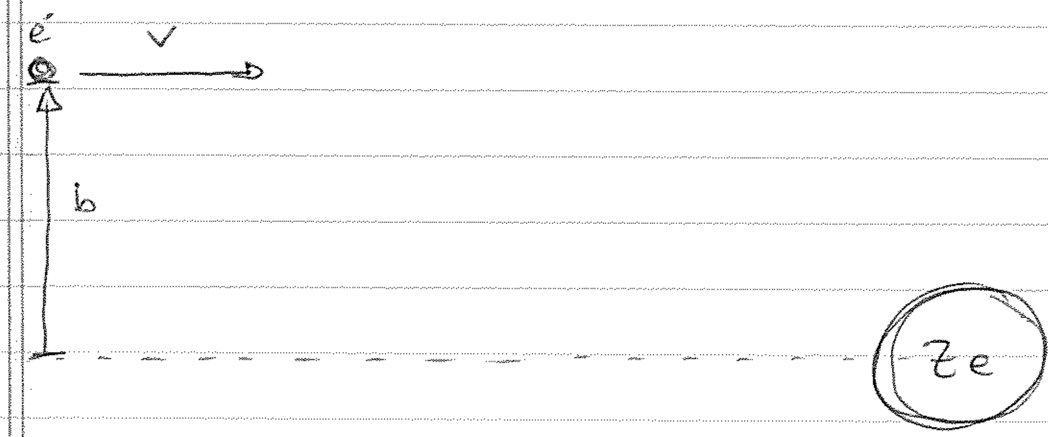
THIS UNIFORM MOTION WILL SERVE AS THE PREDOMINANT MOTION/RESPONSE. THE SECONDARY INFLUENCE WILL BE THE FORCE ~~[illegible]~~ $\vec{F}(\vec{x})$ EXERTED BY THE SCATTERER.

1Aii SUPPOSE A FORCE $\vec{F}(\vec{x})$ IS EXERTED ON A PARTICLE AT POSITION $\vec{x}_0$ FOR A VERY SHORT TIME Δt. BY HOW MUCH IS THE PARTICLE'S VELOCITY ALTERED? [BAD HINT: PARTICLE'S MASS IS m]

IF THE FORCE IS WEAK AND THE PARTICLE IS MOVING FAST, THEN THESE TINY IMPULSES [I.E. FORCES BEING APPLIED OVER SHORT TIME] DO NOT SIGNIFICANTLY CHANGE THE PARTICLE'S VELOCITY, ~~AND~~ SO THE PARTICLE ESSENTIALLY CONTINUES ALONG ITS PATH OF UNIFORM MOTION.

(1A iii) DERIVE AN EXPRESSION TO APPROXIMATE THE NET CHANGE IN VELOCITY OF A PARTICLE w/ MASS m WITH LARGE VELOCITY $\vec{v}$ ~~MOVING~~ APPROACHING FROM A FARAWAY POINT x_0 AND MOVING ~~THROUGH~~ ~~PAST~~ THROUGH A WEAK & LOCALIZED POTENTIAL $V(\vec{x})$?
(WHAT IS MEANT BY LARGE VELOCITY? WEAK POTENTIAL? COMPARED TO WHAT?)

(1A iv) USE THIS RESULT TO CALCULATE THE ANGLE THAT A NUCLEUS OF CHARGE Ze WOULD SCATTER AN ELECTRON APPROACHING WITH AN IMPACT PARAMETER b:



(HINT : $\int_{-\infty}^{\infty} dz \frac{1}{\left[1+z^2\right]^{3/2}} = 2$)

WHY IS THERE NO CHANGE IN THE VELOCITY IN THE DIRECTION OF THE ~~par~~ ELECTRON'S INITIAL VELOCITY? PROVIDE A CONDITION FOR THE VALIDITY OF THIS APPROXIMATE TECHNIQUE IN TERMS OF COMPARISON OF TWO ENERGIES.

1B THE BORN APPROXIMATION IN WAVE SCATTERING

THE FOLLOWING APPLIES TO ANY <u>WAVE</u>; THAT IS, ANYTHING OBEYING A WAVE EQUATION:

$$\nabla^2 \psi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi = 0$$

ONE SUCH SOLUTION IS:

$$\psi(\vec{x},t) = \cancel{\psi_0 \exp\left[i\left(\omega t - \frac{\omega}{c} r\right)\right]}$$

$$\cancel{\psi_0 \exp\left[i\left(\omega t - \frac{\omega}{c}\left|\vec{x} - \vec{x}_0\right.\right.\right.}$$

$$\frac{\psi_0 \exp\left[i\left(\omega t - \frac{\omega}{c}\left|\vec{x} - \vec{x}_0\right|\right)\right]}{\left|\vec{x} - \vec{x}_0\right|}$$

WHICH REPRESENTS A POINT SOURCE RADIATING AT A FREQUENCY $\frac{\omega}{2\pi}$ AT A POINT $\underline{x}_0$

(1Bi) ~~SHOW THAT THE CLOSE VICINITY OF AN OF A POINT~~ SHOW THAT THE CLOSE VICINITY OF A POINT $\underline{x}$ FAR AWAY FROM A SOURCE AT $\underline{x}_0$ LOOKS LIKE A PLANE WAVE. WHAT IS ITS DIRECTION? WAVELENGTH? PICTORIAL PROOF ACCEPTABLE.

(HINT: A PLANE WAVE IS OF THE FORM:

$$A \exp[i(\omega t - \underline{k}\cdot\underline{x})]$$

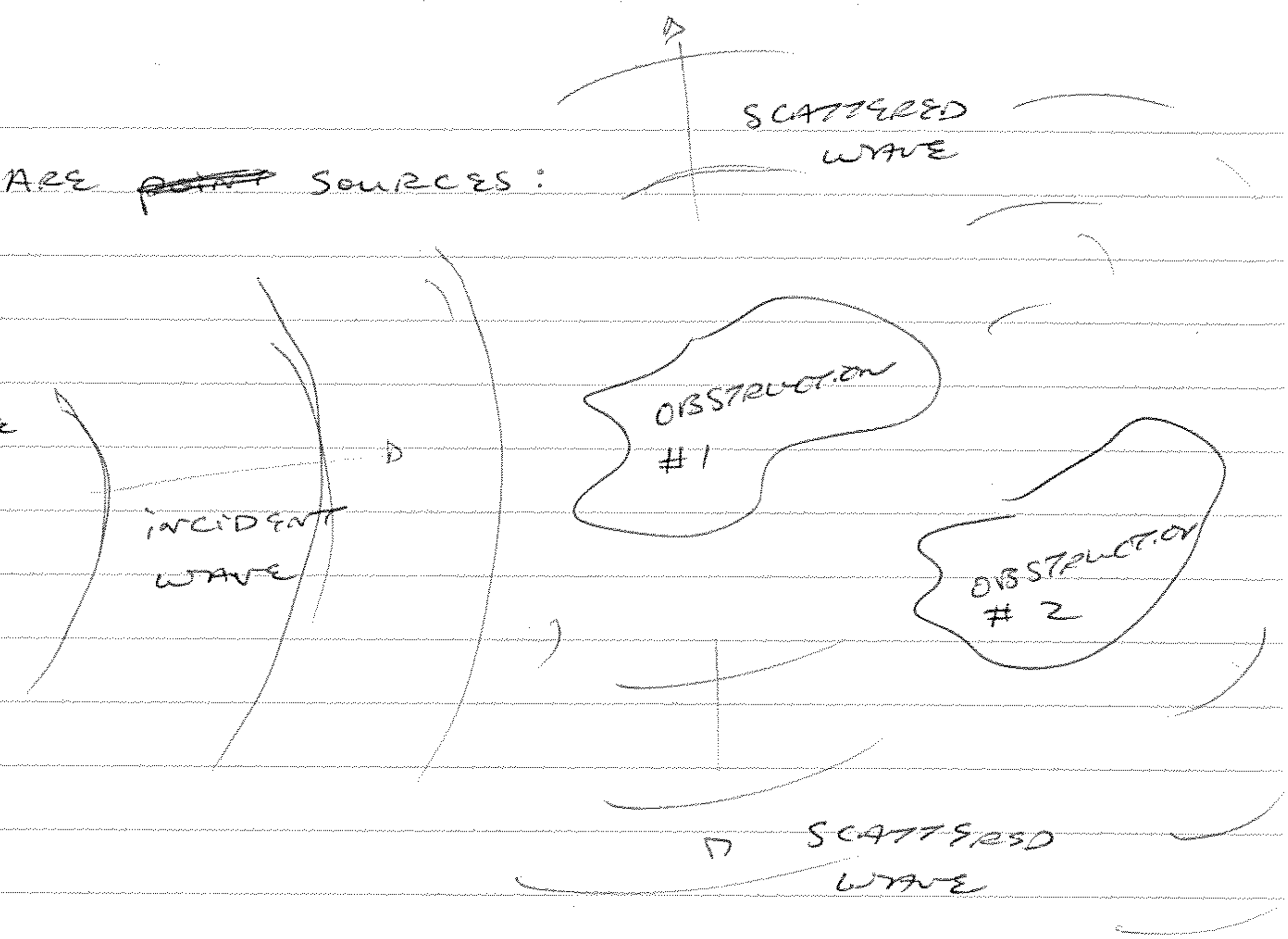
WHERE $\frac{\underline{k}}{|k|}$ IS THE DIRECTION AND $2\pi/k$ IS THE WAVELENGTH)

(HINT: ~~SOLVE~~ CALCULATE WAVE AMPLITUDE FOR POINTS A SHORT DISPLACEMENT Δx FROM $\underline{x}$)

KEEP THIS RESULT IN YOUR POCKET.

NOW CONSIDER OBSTRUCTIONS PLACED IN THE WAY OF THIS RADIATING SOURCE. THESE OBSTRUCTIONS WILL SCATTER ENERGY AWAY, ACTING AS THOUGH THEY THEMSELVES

ARE ~~point~~ SOURCES:



WAVES SCATTERED FROM ONE OBSTRUCTION WILL RUN INTO OTHER OBSTRUCTIONS, CREATING ANOTHER SCATTERED WAVE, SO ON & SO ON.

THE BORN APPROXIMATION ASSUMES SIMPLY THAT THE SCATTERED WAVES ARE SO WEAK THAT THEY CREATE NEGLIGIBLE SECONDARY SCATTERING.

SO IN THE GENERAL FRAMEWORK OF PERT. THEORY, THE WAVE EQUATION IS THE PRIMARY INFLUENCE, PROPAGATING RADIAL WAVES AWAY FROM SOURCES. THESE RADIAL WAVES THEN ~~excite scattered waves~~

TRAVEL THROUGH OBSTRUCTIONS [THE SECONDARY INFLUENCE], CREATING SCATTERED WAVES [THE SECONDARY EFFECT].

~~SUPPOSE A WAVE WITH AMPLITUDE $\psi(\vec{x}, t)$ GENERATES A SCATTERED WAVE AT [illegible] $\phi(\vec{x}, t; \vec{x}')$ AT AN OBSTRUCTION OF TINY VOLUME dx^3 @ $\vec{x}'$ ACCORDING TO THE RULE:~~

~~$$\phi(\vec{x}, t; \vec{x}')$$~~

~~$$= V(\vec{x}')\, dx^3 \cdot \exp(i($$~~

~~IN THE BORN APPROXIM~~ TINY OBSTRUCTIONS GENERATE SPHERICAL WAVES IN PROPORTION TO THE AMOUNT THEY ARE EXCITED. I.E. AN OBSTRUCTION EXCITED BY AN INCIDENT WAVE $\psi(\vec{x}', t')$ AT A POINT $\vec{x}'$ & TIME t' WILL GENERATE A SCATTERED WAVE ~~[illegible]~~ $\Delta\phi(\vec{x}, t)$ AT A POINT $\vec{x}$ & TIME t ACCORDING TO THE RULE:

$$\Delta\phi(\vec{x}, t) = \cancel{[illegible]}\ \frac{\Delta V \cdot A \cdot \psi\left(\vec{x}', t - \frac{|\vec{x} - \vec{x}'|}{c}\right)}{|\vec{x} - \vec{x}'|}$$

WHERE ΔV IS THE TINY ~~[illegible]~~ VOLUME OCCUPIED BY THE OBSTRUCTION & A IS A WEIGHTING FACTOR INDICATING THE STRENGTH OF THE OBSTRUCTI

SO THE SCATTERED WAVE AMPLITUDE A DISTANCE r FROM THE OBSTRUCTION IS SIMPLY PROPORTIONAL TO THE EXCITING WAVE'S AMPLITUDE AT THE OBSTRUCTION AT A TIME $\frac{r}{c}$ EARLIER, SINCE THE WAVE CAN ONLY PROPAGATE AT A SPEED C.

1 Bii ~~CALC~~ WRITE DOWN EXPRESSION FOR THE SCATTERED WAVE (IN THE BORN APPROXIMATION) FOR AN EXTENDED OBSTRUCTION DESCRIBED BY A WEIGHTING FUNCTION $A(\vec{x}')$ DESCRIBING THE STRENGTH OF THE OBSTRUCTION AT EACH POINT. GIVE THE ~~ANSWER AS~~ ANSWER AS A FUNCTION OF THE OBSERVATION POINT / TIME $\vec{x}$, t, AND THE INCIDENT / EXCITING WAVE $\psi(\vec{x}', t')$

(HINT: SPLIT THE EXTENDED OBSTRUCTION ~~AS~~ INTO A BUNCH [INFINITE #] OF TINY CHUNKS)

I.E. $\phi(\vec{x}, t) = \,?$

BONUS!

1 Biii USE THIS EQUATION TO SOLVE FOR THE SCATTERED WAVE FROM A CUBIC ARRAY (N X N X N) OF DELTA FUNCTION OBSTRUCTIONS:

I.E.

$$A(\vec{x}') = A_0 \sum_{lmn=0}^{N} \delta(x' - a\cdot l)\,\delta(y' - a\cdot m) \cdot \delta(z' - a\cdot n)$$

MORE ON NEXT

~~Simplify~~

ASSUME:

OF AMPLITUDE z_0

- INCIDENT WAVE GENERATED FROM SOURCE FAR AWAY FROM ARRAY. POINT
- INTERESTED IN SCATTERED WAVE AMPLITUDE FAR AWAY FROM ARRAY.
- EXPRESS ANSWER IN TERMS OF:

$\vec{r}\,'$: ~~VECTOR JOINING~~ LOCATION OF SOURCE

$\vec{r}$: ~~OBSERVATIO~~ OBSERVATION POINT OF SCATTERED WAVE

ω : SOURCE FREQUENCY

NOTE THAT ~~OF~~ ARRAY IS LOCATED ~~NEAR~~ AROUND $\vec{x} = \vec{0}$

I.E. ONLY SHOW SPATIAL DEPENDENCE

1C GIVE ANOTHER EXAMPLE OF PERTURBATION THEORY APPLIED TO PHYSICS.

(2) QUARTIC PERTURBATION OF HARMONIC OSCILLATOR

SUPPOSE A QUARTIC PERTURBATION ~~IS~~ APPLIED TO A SIMPLE ~~HARMONIC~~ OSCILLATOR OF FREQUENCY ω_0

$$H_0 = \frac{1}{2}\frac{p^2}{m} + \frac{1}{2} m\omega^2 x^2$$

$$H' = \alpha x^4$$

LET'S REARRANGE THINGS TO REDUCE THE ~~AMOUN~~ NUMBER OF CONSTANTS WE CARRY THROUGH THE CALCULATION

(2A) NORMALIZE THE UNPERTURBED SCHRODINGER EQUATION SO THAT THE HAMILTONIAN, IN UNITS OF $\hbar\omega$, IS: $\frac{1}{2}(\tilde{p}^2 + \tilde{x}^2) = \tilde{H}_0$

WHERE $\tilde{p}$ & $\tilde{x}$ ARE UNITLESS AND PROPORTIONAL TO p & x, RESPECTIVELY. WHAT ~~NATURAL~~ ARE THESE CONSTANTS OF PROPORTIONALITY $p_0 = p/\tilde{p}$, $x_0 = x/\tilde{x}$?

(2B) EXPRESS THE PERTURBATION IN THESE NORMALIZED UNITS. WRITE ~~THE UNITLESS~~ ~~PARAM~~ AS $\tilde{H}' = \beta \tilde{x}^4$. WHAT IS β? WHAT ARE ITS UNITS?

2C WHAT IS $\tilde{x}$ EXPRESSED IN ~~UNITS~~ TERMS OF a_+ AND a_- [~~CREAT~~ RAISING AND LOWERING OPERATORS]?
EXPRESS $\tilde{H}'$ IN TERMS OF THESE OPERATORS. THERE SHOULD BE SIXTEEN TERMS, ALL EXPANDED.

2D LET'S CALCULATE THE FIRST ORDER SHIFT IN GROUND STATE ENERGY DUE TO OUR PERTURBATION:

$$\langle 0 | H' | 0 \rangle$$

$$= \frac{\hbar\omega}{4}\beta \langle 0 | (a_+ + a_-)^4 | 0 \rangle$$

WHAT RULES CAN YOU COME UP WITH TO ELIMINATE TERMS IN THE HAMILTONIAN?

WHAT IS LEFT AFTER ELIMINATING?

2E WHAT IS YOUR ANSWER? WHAT CONDITION DO WE PLACE ON α ~~tha~~ IN ORDER TO JUSTIFY APPLICATION OF PERT. THEORY?